International Journal for Research in Applied Sciences and Biotechnology

ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)

www.ijrasb.com

https://doi.org/10.31033/ijrasb.9.1.1

Finite Integral Involving the Modified Generalized I-Function of Two Variables, Generalized Extented Hurwitz's Zeta Function of Two Variables and Exponential Function

Frédéric Ayant[1] and Prvindra Kumar[2]
[1]Teacher in High School, FRANCE
[2]Department of Mathematics, D. J. College, Baraut – 250, INDIA

[2]Corresponding Author: prvindradjc@gmail.com

ABSTRACT

In the present paper, we evaluate the general finite integral involving the exponential function, generalized Hurwitz's-Lerch zeta function of two variables and the modified of generalized I-function of two variables. At the end, we shall see several corollaries and remarks.

Keywords- Modified generalized I-function of two variables, generalized I-function of two variables, generalized modified H-function of two variables, generalized modified Meijer-function of two variables, I-function of two variables, H-function of two variables, Meijer-function of two variables, double Mellin-Barnes integrals contour, finite integral, generalized extented Hurwitz's Zeta function of two variables.

2010 Mathematics Subject Classification: 33C05, 33C60.

I. INTRODUCTION AND PRELIMINARIES

Kumari et al. [14] have defined the I-function of two variables. On the other hand, Prasad and Prasad [18] have studied the modified generalized H-function of two variables. Recently Singh and Kumar [22] have worked about the modified of generalized I-function of two variables. This function is an extension of the I-function of two variables and modified of generalized H-function of two variables. In this paper, first, we define the modified generalized I-function of two variables. Then we calculate the finite integral involving this function. At the last section, we will see several cases and remarks. The double Mellin-Barnes integrals contour occurring in this paper will be referred to as the generalized modified I-function of two variables throughout our present study and will be defined and represented as follows:

We have

$$I(z_1,z_2)=I^{m_1,n_1,m_2,n_2,m_3,n_3,m_4,n_4}_{p_1,q_1,p_2,q_2;p_3,q_3;p_4,q_4}\left(\begin{array}{c|c} \mathrm{z}_1 & \{(\mathrm{a}_i;\alpha_i,A_i;\mathbf{A}_i)\}_{1,p_1}:\{(c_i;\gamma_i,C_i;\mathbf{C}_i)\}_{1,p_2}:\{(e_i;E_i;\mathbf{E}_i)\}_{1,p_3},\{(g_i;G_i;\mathbf{G}_i)\}_{1,p_4} \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_2 & \{(\mathrm{b}_i;\beta_i,B_i;\mathbf{B}_i)\}_{1,q_1}:\{(d_i;\delta_i,D_i;\mathbf{D}_i)\}_{1,q_2}:\{(f_i;F_i;\mathbf{F}_i)\}_{1,q_3},\{(h_i;H_i;\mathbf{H}_i)\}_{1,q_4} \end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\theta(s,t)\psi(s)\phi(t)z_1^s z_2^t\mathrm{d}s\mathrm{d}t \tag{1.1}$$

where

$$\theta(s,t)=\frac{\prod_{j=1}^{m_1}\Gamma^{\mathbf{B}_j}(b_j-\beta_j s-B_j t)\prod_{j=1}^{n_1}\Gamma^{\mathbf{A}_j}(1-a_j+\alpha_j s+A_j t)\prod_{j=1}^{m_2}\Gamma^{\mathbf{D}_j}(d_j-\delta_j s+D_j t)}{\prod_{j=m_1+1}^{q_1}\Gamma^{\mathbf{B}_j}(1-b_j+\beta_j s+B_j t)\prod_{j=n_1+1}^{p_1}\Gamma^{\mathbf{A}_j}(a_j-\alpha_j s-A_j t)\prod_{j=m_2+1}^{q_2}\Gamma^{\mathbf{D}_j}(1-d_j+\delta_j s-D_j t)}$$

$$\frac{\prod_{j=1}^{n_2}\Gamma^{\mathbf{C}_j}(1-c_j+\gamma_j s-C_j t)}{\prod_{j=1}^{n_2}\Gamma^{\mathbf{C}_j}(c_j-\gamma_j s+C_j t)} \tag{1.2}$$

$$\psi(s)=\frac{\prod_{j=1}^{m_3}\Gamma^{\mathbf{F}_j}(f_j-F_i s)\prod_{j=1}^{n_3}\Gamma^{\mathbf{E}_j}(1-e_j+E_j s)}{\prod_{j=m_3+1}^{q_3}\Gamma^{\mathbf{F}_j}(1-f_j+F_j s)\prod_{j=n_3+1}^{p_3}\Gamma^{\mathbf{E}_j}(e_j-E_j s)} \tag{1.3}$$

and

$$\phi(t) = \frac{\prod_{j=1}^{m_4} \Gamma^{\mathbf{H}_j}(h_j - H_j t) \prod_{j=1}^{n_4} \Gamma^{\mathbf{G}_j}(1 - g_j + G_j t)}{\prod_{j=m_4+1}^{q_4} \Gamma^{\mathbf{H}_j}(1 - h_j + H_j t) \prod_{j=n_4+1}^{p_4} \Gamma^{\mathbf{G}_j}(g_j - G_j t)} \tag{1.4}$$

where $z_1$and z_2 are not zero and an empty product is interpreted as unity. Also $m_i, n_i, p_i, q_i (i = 1, 2, 3, 4)$ are all positive integers such that $0 \leqslant m_i \leqslant q_i ; 0 \leqslant n_i \leqslant p_i (i = 1, 2, 3, 4)$. The letters $\alpha, \beta, \gamma, \delta, A, B, C, D, E, F, G, H$ and $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$ are all positive numbers and the letters a, b, c, d, e, f, g, h are complex numbers.
the definition of modified generalized I-function of two variables given above will however, have a meaning even if some of these quantities are zero. The contour L_1 is in the s-plane and runs from $-\omega\infty$ to $+\omega\infty$ with loops, if necessary, to ensure that the poles of $\Gamma^{\mathbf{B}_\mathbf{i}}(b_j - \beta_i s - B_i t)(i = 1, \cdots, m_1)$, $\Gamma^{\mathbf{D}_\mathbf{i}}(d_i - \delta_i s + D_i t)(i = 1, \cdots, m_2)$ and $\Gamma^{\mathbf{F}_\mathbf{j}}(f_j - F_j s)(j = 1, \cdots, m_3)$, are to the right and all the poles of $\Gamma^{\mathbf{A}_\mathbf{j}}(1 - a_j + \alpha_j s + A_j t)(j = 1, \cdots, n_1)$, $\Gamma^{\mathbf{E}_\mathbf{j}}(1 - e_j + E_j s)$,$(j = 1, \cdots, n_3)$ and $\Gamma^{\mathbf{C}_\mathbf{j}}(1 - c_j + \gamma_j s - C_j t)(j = 1, \cdots, n_2)$ lie to the left of L_1. The contour L_2 is in the t-plane and runs from $-\omega\infty$ to $+\omega\infty$ with loops, if necessary, to ensure that the poles of $\Gamma^{\mathbf{B}_\mathbf{i}}(b_j - \beta_i s - B_i t)(i = 1, \cdots, m_1)$, $\Gamma^{\mathbf{H}_\mathbf{j}}(h_j - H_j t)(j = 1, \cdots, m_4)$ $\Gamma^{\mathbf{C}_\mathbf{j}}(1 - c_j + \gamma_j s - C_j t)(j = 1, \cdots, n_2)$ are to the right and all the poles of $\Gamma^{\mathbf{A}_\mathbf{j}}(1 - a_j + \alpha_j s + A_j t)(j = 1, \cdots, n_1)$, $\Gamma^{\mathbf{D}_\mathbf{j}}(d_j - \delta_j s + D_j t)(j = 1, \cdots, m_2)$ and $\Gamma^{\mathbf{G}_\mathbf{j}}(1 - g_j + G_j t)(j = 1, \cdots, n_4)$ lie to the left of L_2. The poles of the integrand are assumed to be simple.

The function defined by the equation (3.1) is analytic function of z_1 and z_2 if

$$\sum_{i=1}^{p_1} \mathbf{A}_i \alpha_i + \sum_{i=1}^{p_2} \mathbf{C}_i \gamma_i + \sum_{i=1}^{p_3} \mathbf{E}_i E_i < \sum_{i=1}^{q_1} \mathbf{B}_i \beta_i + \sum_{i=1}^{q_2} \mathbf{D}_i \delta_i + \sum_{i=1}^{q_3} \mathbf{F}_i F_i \tag{1.5}$$

$$\sum_{i=1}^{p_1} \mathbf{A}_i A_i + \sum_{i=1}^{p_2} \mathbf{D}_i C_i + \sum_{i=1}^{p_4} \mathbf{G}_i G_i < \sum_{i=1}^{q_1} \mathbf{B}_i B_i + \sum_{i=1}^{q_2} \mathbf{C}_i C_i + \sum_{i=1}^{q_4} \mathbf{H}_i H_i \tag{1.6}$$

The integral (3.1) is absolutly convergent if $|argz_1| < \frac{1}{2}U\pi, |argz_2| < \frac{1}{2}V\pi$ where

$$U = \sum_{j=1}^{n_1} \mathbf{A}_j \alpha_j - \sum_{j=n_1+1}^{p_1} \mathbf{A}_j \alpha_j + \sum_{j=1}^{m_1} \beta_j \mathbf{B}_j - \sum_{j=m_1+1}^{q_1} \beta_j \mathbf{B}_j + \sum_{j=1}^{n_2} \mathbf{C}_j \gamma_j - \sum_{j=n_2+1}^{p_2} \mathbf{C}_j \gamma_j + \sum_{j=1}^{m_2} \mathbf{D}_j \delta_j$$

$$- \sum_{j=m_2+1}^{q_2} \mathbf{D}_j \delta_j + \sum_{j=1}^{n_3} \mathbf{E}_j E_j - \sum_{j=n_3+1}^{p_3} \mathbf{E}_j E_j + \sum_{j=1}^{m_3} \mathbf{F}_j F_j - \sum_{j=m_3+1}^{q_3} \mathbf{F}_j F_j > 0 \tag{1.7}$$

$$V = \sum_{j=1}^{n_1} \mathbf{A}_j A_j - \sum_{j=n_1+1}^{p_1} \mathbf{A}_j A_j + \sum_{j=1}^{m_1} B_j \mathbf{B}_j - \sum_{j=m_1+1}^{q_1} B_j \mathbf{B}_j + \sum_{j=1}^{n_2} \mathbf{C}_j C_j - \sum_{j=n_2+1}^{p_2} \mathbf{C}_j C_j + \sum_{j=1}^{m_2} \mathbf{D}_j D_j$$

$$- \sum_{j=m_2+1}^{q_2} \mathbf{D}_j \delta_j + \sum_{j=1}^{n_4} \mathbf{G}_j G_j - \sum_{j=n_4+1}^{p_4} \mathbf{G}_j G_j + \sum_{j=1}^{m_4} \mathbf{H}_j H_j - \sum_{j=m_4+1}^{q_4} \mathbf{H}_j H_j > 0 \tag{1.8}$$

We may establish the the asymptotic behavior in the following convenient form, see B.L.J. Braaksma [7].

$$I(z_1, z_2) = 0(\, |z_1|^{\alpha_1}, |z_2|^{\alpha_2} \,) \, , max(\, |z_1|, |z_2| \,) \to 0$$

$$I(z_1, z_2) = 0(\, |z_1|^{\beta_1}, |z_2|^{\beta_2} \,) \, , min(\, |z_1|, |z_2| \,) \to \infty \, :$$

$\alpha_1 = \min_{1 \leqslant j \leqslant m_3} Re \left[\mathbf{F}_j \left(\frac{f_j}{F_j} \right) \right]$ and $\alpha_2 = \min_{1 \leqslant j \leqslant m_4} Re \left[\mathbf{H}_j \left(\frac{h_j}{H_j} \right) \right]$

ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)

https://doi.org/10.31033/ijrasb.9.1.1

$$\beta_1 = \max_{1\leqslant j\leqslant n_2} Re\left[\mathbf{E}_j\left(\frac{e_j-1}{E_j}\right)\right] \text{ and } \beta_2 = \max_{1\leqslant j\leqslant n_4} Re\left[\mathbf{G}_j\left(\frac{g_j-1}{G_j}\right)\right]$$

Now, we cite several generalized Hurwitz's Zeta function. The estended Hurwitz's Zeta function $\phi(x,s,a)$ is defined by [9,24] :

$$\phi(z,s,a) = \sum_{n=0}^{\infty} \frac{z^n}{(a+s)^n} \tag{1.9}$$

The authors Bin-Saad, Garg et al. [5,6] and other workers have defined a generalization of the Zeta-function by :

$$\phi^*(z,s,a) = \sum_{n=0}^{\infty} \frac{(\mu)_n z^n}{(a+s)^n n!} \tag{1.10}$$

where$\mu \in \mathbb{C}, a \in \mathbb{C} \setminus \mathbb{Z}^0; s \in \mathbb{C}$ when $|z| < 1, Re(s-\mu) > 1$ when $|z| = 1$.

We note

$$\mathrm{A}^*(n,s,a) = \frac{(\mu)_n}{(a+s)^n n!} \tag{1.11}$$

We define a generalized of extended Hurwitz's Zeta function of two variables. First time, we define a generalized Exton function of two variables [10], Exton function of two variables [11] and Kampe de Feriet function [25,26], then we will give generalized extented Hurwitz's Zeta function of two variables by using the notations about the generalized hypergeometric function of two variables defined above.

The generalized Exton function is defined by [10,page 339, Eq. 13], it's note $\mathrm{H}_{E:G;M;N}^{A:B;C;D}()$ with

$$\mathrm{H}_{E:G;M;N}^{A:B;C;D}(x,y) = H_{E:G;M;N}^{A:B;C;D}\left[\begin{array}{c|c} (\mathrm{a}_A):(b_B);(c_C);(d_D) & \\ \cdots & \mathrm{x,y} \\ (\mathrm{e}_E):(g_G);(m_M);(n_N) & \end{array}\right] = \sum_{i,j=0}^{\infty} \frac{[a_A]_{2i+j}[b_B]_{i+j}[c_C]_i[d_D]_j}{[e_E]_{2i+j}[g_G]_{i+j}[m_M]_i[n_N]_j} \frac{x^i}{i!}\frac{y^j}{j!} \tag{1.12}$$

where $0 < |x|, |y| < 1$.

If $\mathrm{B} = \mathrm{G} = 0$ in (1.20), then the above function reduces to the double hypergeometric function defined Exton [11, p.137, Eq.12]

$$\mathrm{X}_{E:M;N}^{A:C;D}(x,y) = X_{E:M;N}^{A:C;D}\left[\begin{array}{c|c} (\mathrm{a}_A):(c_C);(d_D) & \\ \cdots & \mathrm{x,y} \\ (\mathrm{e}_E):(m_M);(n_N) & \end{array}\right] = \sum_{i,j=0}^{\infty} \frac{[a_A]_{2i+j}[c_C]_i[d_D]_j}{[e_E]_{2i+j}[m_M]_i[n_N]_j} \frac{x^i}{i!}\frac{y^j}{j!} \tag{1.13}$$

where $0 < |x|, |y| < 1$.

If $\mathrm{A} = \mathrm{E} = 0$ in (1.20), then the above function reduces to the double hypergeometric function defined Kampe de Fériet, see Srivastava, Srivastava and Pathan, [25, p. 423 (26)] and [26] respectively.

$$\mathrm{F}_{G;M;N}^{B;C;D}(x,y) = H_{0,G;M;N}^{0:B;C;D}\left[\begin{array}{c|c} (\mathrm{a}_A):(b_B);(c_C);(d_D) & \\ \cdots & \mathrm{x,y} \\ (\mathrm{e}_E):(g_G);(m_M);(n_N) & \end{array}\right] = \sum_{i,j=0}^{\infty} \frac{[b_B]_{i+j}[c_C]_i[d_D]_j}{[g_G]_{i+j}[m_M]_i[n_N]_j} \frac{x^i}{i!}\frac{y^j}{j!} \tag{1.14}$$

where $0 < |x|, |y| < 1$.

Recently Pathan and Daman [12] introduced a generalization in terms of double series representation :

$$\Phi_{\alpha;\beta;\gamma:\lambda,\mu;\upsilon}^{p,q}(z,t,s,a) = \sum_{i,j=0}^{\infty} \frac{(\alpha)_i(\beta)_i(\lambda)_j(\upsilon)_j}{(\mu)_i(\gamma)_i(\upsilon)_j(a+pi+qj)^s} \frac{z^i}{i!}\frac{t^j}{j!} \tag{1.15}$$

We will note

ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)

https://doi.org/10.31033/ijrasb.9.1.1

$$\mathrm{A}^{p,q}_{\alpha;\beta;\gamma:\lambda,\mu;\upsilon}(s,a)=\frac{(\alpha)_i(\beta)_i(\lambda)_j(\upsilon)_j}{(\mu)_i(\gamma)_i(\upsilon)_j(a+pi+qj)^s}\frac{1}{i!}\frac{1}{j!} \tag{1.16}$$

where $\gamma,\upsilon,a\notin\{0,-1,-2,\cdots\}, s\in\mathbb{C},p,q,Re(s)>0$ when $|z|,|t|<1$ and $\mathrm{Re}(\gamma+\upsilon+s-\alpha-\beta-\lambda-\mu)>0$ when $|z|=|t|=1$.

Pathan et al. [16] give a extension of the generalized Hurwitz-Lerch zeta-function of two variables defined by

$${}_{p,q}\Phi^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(z,t,s,a)=\sum_{i,j=0}^{\infty}\frac{[\alpha_H]_{2i+j}[\beta_J]_{i+j}[\gamma_K]_i[\lambda_L]_j}{[\mu_N]_{2i+j}[\upsilon_R]_{i+j}[\rho_V]_i[\sigma_W]_j}\frac{z^it^j}{i!j!(a+pi+qj)^s} \tag{1.17}$$

under the conditions named (E)

$\alpha_H(H=1,\cdots,h),\beta_J(J=1,\cdots,k),\gamma_K(K=1,\cdots,l),\lambda_L(L=1,\cdots,m)$ are complex numbers $\mu_N(N=1,\cdots,n)$,

$\upsilon_R(R=1,\cdots,r)$, $\rho_V(V=1,\cdots,\upsilon),\sigma_W(T=1,\cdots,w),a\in\mathbb{C}\setminus\mathbb{Z}_0^-,p,q>0,s\in\mathbb{C},Re(s)>0$ when $|z|,|t|<1$ and

$Re\left(\sum_{N=1}^{n}\mu_n+\sum_{R=1}^{r}\upsilon_R+\sum_{V=1}^{\upsilon}\rho_s+\sum_{W=1}^{w}\sigma_T+s-\sum_{H=1}^{h}\alpha_H-\sum_{K=1}^{k}\beta_K-\sum_{K=1}^{l}\gamma_K-\sum_{L=1}^{m}\lambda_L\right)>0$ when $|z|=|t|=1$.

We will pose :

$${}_{p,q}A^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(s,a)=\frac{[\alpha_H]_{2i+j}[\beta_J]_{i+j}[\gamma_K]_i[\lambda_l]_j}{[\mu_N]_{2i+j}[\upsilon_R]_{i+j}[\rho_V]_i[\sigma]_W}\frac{z^it^j}{i!j!(a+pi+qj)^s} \tag{1.18}$$

Let $\mathrm{J}=\mathrm{R}=0$, we get

$${}_{p,q}\Phi^{\alpha:\gamma;\lambda,\mu:\sigma}_{H:K;L;M,N:V;W}(z,t,s,a)=\sum_{i,j=0}^{\infty}\frac{[\alpha_H]_{2i+j}[\gamma_K]_i[\lambda_l]_j}{[\mu_N]_{2i+j}[\rho_V]_i[\sigma_W]_j}\frac{z^it^j}{i!j!(a+pi+qj)^s} \tag{1.19}$$

under the conditions named (E')

$\alpha_H(I=1,\cdots,h),\gamma_K(K=1,\cdots,l),\lambda_L(L=1,\cdots,m)$ are complex numbers $\mu_N(N=1,\cdots,n)$, $\rho_V(V=1,\cdots,\upsilon)$

$\sigma_W(T=1,\cdots,w),a\in\mathbb{C}\setminus\mathbb{Z}_0^-,p,q>0,s\in\mathbb{C},Re(s)>0$ when $|z|,|t|<1$ and

$Re\left(\sum_{N=1}^{n}\mu_n+\sum_{S=1}^{\upsilon}\rho_s+\sum_{T=1}^{W}\sigma_T+s-\sum_{H=1}^{h}\alpha_H-\sum_{K=1}^{l}\gamma_K-\sum_{L=1}^{m}\lambda_L\right)>0$ when $|z|=|t|=1$.

We will note

$${}_{p,q}A^{\alpha:\gamma;\lambda,\mu:\sigma}_{H:K;L;M,N:V;W}(s,a)=\frac{[\alpha_H]_{2i+j}[\gamma_K]_i[\lambda_l]_j}{[\mu_N]_{2i+j}[\rho_V]_i[\sigma_W]_j}\frac{1}{i!j!(a+pi+qj)^s} \tag{1.20}$$

We suppose $\mathrm{H}=\mathrm{N}=0$, we have

$${}_{p,q}\Phi^{\beta;\gamma;\lambda:\upsilon;\rho;\sigma}_{K;L;M:R;\upsilon;w}(z,t,s,a)=\sum_{i,j=0}^{\infty}\frac{[\beta_J]_{i+j}[\gamma_K]_i[\lambda_l]_j}{[\upsilon_R]_{i+j}[\rho_V]_i[\sigma_W]_j}\frac{z^it^j}{i!j!(a+pi+qj)^s} \tag{1.21}$$

under the conditions named (E'')

$\beta_J(J=1,\cdots,k),\gamma_K(K=1,\cdots,l)$, are complex numbers $\mu_N(N=1,\cdots,n),\upsilon_R(R=1,\cdots,r)$, $\rho_V(V=1,\cdots,\upsilon)$

$\sigma_W(T=1,\cdots,w),a\in\mathbb{C}\setminus\mathbb{Z}_0^-,p,q>0,s\in\mathbb{C},Re(s)>0$ and we have the following relation

$Re\left(\sum_{R=1}^{r}\upsilon_R+\sum_{V=1}^{\upsilon}\rho_s+\sum_{W=1}^{w}\sigma_T+s-\sum_{K=1}^{k}\beta_K-\sum_{K=1}^{l}\gamma_K-\sum_{L=1}^{m}\lambda_L\right)>0$ when $|z|=|t|=1$.

We will use the following formula.

$${}_{p,q}A^{\beta;\gamma;\lambda:\upsilon;\rho;\sigma}_{K;L;M:R;\upsilon;w}(s,a)=\frac{[\beta_J]_{i+j}[\gamma_K]_i[\lambda_l]_j}{[\upsilon_R]_{i+j}[\rho_V]_i[\sigma_W]_j}\frac{1}{i!j!(a+pi+qj)^s} \tag{1.22}$$

ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)

https://doi.org/10.31033/ijrasb.9.1.1

II. REQUIRED INTEGRAL

In this section, we give a general finite integral, see Brychkov ([8], 4.2.5 Eq.123 page 147).

Lemma

$$\int_0^a x^s (a-x)^{s+\frac{1}{2}} e^{b\sqrt{x(a-x)}}\mathrm{d}x = 2^{-2s-1}\sqrt{\pi}a^{2s+\frac{3}{2}}\frac{\Gamma(2s+2)}{\Gamma\left(2s+\frac{5}{2}\right)}{}_1F_1\left(\begin{array}{c} 2s+2 \\ \cdot\cdot\cdot \\ 2s+\frac{5}{2}\end{array}\middle| \frac{ab}{2}\right) \tag{2.1}$$

where $Re(s) > -1, |arg(2-ab)| < \pi,\ a > 0$.

III. MAIN INTEGRAL

In this section, we study a generalization of the finite integral involving the modified generalized I-function of two variables and the de he generalized Hurwitz-Lerch zeta-function of two variables defined by Pathan et al. [13].

Theorem

$$\int_0^{a'} x^u (a'-x)^{u+\frac{1}{2}} e^{b\sqrt{x(a'-x)}}\ {}^{p,q}\Phi^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(Zx^A(a'-x)^A Tx^B(a'-x)^B, s, a)$$

$$I(Z_1x^C(a-x)^C, Z_2x^D(a-x)^D)\mathrm{d}x = 2^{-2u-1}\sqrt{\pi}a^{2u+\frac{3}{2}}\sum_{i,j,n'=0}^{\infty}\ {}^{p,q}A^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(s,a)\frac{1}{n'!}\left(\frac{a'b}{2}\right)^{n'}$$

$$\mathrm{Z}^iT^j\, 2^{-2Ai-2Bj}a^{2Ai+2Bj}\, I^{m_1,n_1+1:m_2,n_2:m_3,n_3:m_4,n_4}_{p_1+1,q_1+1:p_2,q_2:p_3,q_3:p_4,q_4}$$

$$\left(\begin{array}{c} \mathrm{Z}_1\left(\frac{a'}{2}\right)^{2C} \\ \cdot \\ \cdot \\ \mathrm{Z}_2\left(\frac{a'}{2}\right)^{2D}\end{array}\middle| \begin{array}{c} \mathbf{A}_1, \{(a_i;\alpha_i,A_i;\mathbf{A}_i)\}_{1,p_1}:\{(c_i;\gamma_i,C_i;\mathbf{C}_i)\}_{1,p_2}:\{(e_i;E_i;\mathbf{E}_i)\}_{1,p_3},\{(g_i;G_i;\mathbf{G}_i)\}_{1,p_4} \\ \cdot \\ \cdot \\ \{(\mathrm{b}_i;\beta_i,B_i;\mathbf{B}_i)\}_{1,q_1},\mathbf{B}_1:\{(d_i;\delta_i,D_i;\mathbf{D}_i)\}_{1,q_2}:\{(f_i;F_i;\mathbf{F}_i)\}_{1,q_3},\{(h_i;H_i;\mathbf{H}_i)\}_{1,q_4}\end{array}\right) \tag{3.1}$$

where

$$\mathbf{A}_1 = (-1-2u-2Ai-2Bj-n';2C,2D;1);\mathbf{B}_1 = \left(-\tfrac{3}{2}-2u-2Ai-2Bj;2C,2D;1\right) \tag{3.2}$$

provided $Re(\mathbf{s}) > -1, |arg(2-ab)| < \pi,\ a > 0$. $-1 < Re(u+Ak+Bl) + 2C\min_{1\leqslant j\leqslant m_3} Re\left(\mathbf{F_j}\frac{f_j}{F_j}\right)$ and $-1 < Re(u+Ak+Bl) + 2D\min_{1\leqslant j\leqslant m_4} Re\left(\mathbf{G_j}\frac{g_j}{G_j}\right)$, $A,B,C,D > 0$ and the conditions (E) are verified and ,$|argz_1| < \frac{1}{2}U\pi, |argz_2| < \frac{1}{2}V\pi$, the numbers U and V are defined respectively by the equations (1.7) and (1.8).respectively,

Proof

To prove the theorem, expressing the generalized Hurwitz-Lerch zeta-function of two variables defined by Pathan et al. [13] in series with the help of (1.17) and the modified generalized I-function of two variables in double Mellin-Barnes integrals contour with the help of (1.1) and interchange the order of the series and integrations which is justifiable due to absolute convergence of the integral involved in the process and collecting the power of , this gives L

$$\mathrm{L} = \int_0^{a'} x^u (a'-x)^{u+\frac{1}{2}} e^{b\sqrt{x(a'-x)}}\ {}^{p,q}\Phi^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(Zx^A(a'-x)^A, Tx^B(a'-x)^B, s, a)$$

$$I(Z_1x^C(a-x)^C, Z_2x^D(a-x)^D)\mathrm{d}x = 2^{-2u-1}\sqrt{\pi}a^{2u+\frac{3}{2}}\sum_{i,j=0}^{\infty}\mathrm{Z}^iT^j\, 2^{-2Bi-2Cj}a^{2Bi+2Cj}$$

$${}^{p,q}A^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(s,a)\ \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\theta(s,t)\psi(s)\phi(t)Z_1^sZ_2^t$$

International Journal for Research in Applied Sciences and Biotechnology

ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)

www.ijrasb.com

https://doi.org/10.31033/ijrasb.9.1.1

$$\int_0^{a'} x^{u+Ai+Bj+Cs+Dt}(a'-x)^{u+\frac{1}{2}-Ai+Bj+Cs+Dt}e^{b\sqrt{x(a'-x)}}\,\mathrm{dx\ ds\ dt} \tag{3.3}$$

We use the lemma, this gives:

$$\int_0^{a'} x^u(a'-x)^{u+\frac{1}{2}}e^{b\sqrt{x(a'-x)}}\ {}^{p,q}\Phi^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(Zx^A(a'-x)^A, Tx^B(a'-x)^B, s, a)$$

$$I(Z_1x^C(a-x)^C, Z_2x^D(a-x)^D)\mathrm{d}x = 2^{-2u-1}\sqrt{\pi}a^{2u+\frac{3}{2}}\sum_{i,j=0}^{\infty}\ {}^{p,q}A^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(s,a)$$

$$\mathrm{Z}^iT^j\ 2^{-2Ai-2Bj}a^{2Ai+2Bj}\ \ \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\theta(s,t)\psi(s)\phi(t)Z_1^sZ_2^t\ 2^{-2As-2Bt}a'^{2As+2Bt}$$

$$\frac{\Gamma(2u+2Ai+2Bj+2Cs+2Dt+2)}{\Gamma(2u+2Ai+2Bj+2Cs+2Dt+\frac{5}{2})}\ {}_1F_1\left(\begin{array}{c|c} \mathrm{2u+2Ai+2Bj+2Cs+2Dt+2} & \\ \cdots & \frac{a'b}{2} \\ \mathrm{2u+2Ai+2Bj+2Cs+2Dt+}\frac{5}{2} & \end{array}\right)\mathrm{ds\ dt} \tag{3.4}$$

We replace the Gauss hypergeometric function by the serie $\sum_{n'=0}^{\infty}$, **(see Slater [23])**, under the **hypothesis**, we can interchanged this serie and the (s,t)- integrals, we have :

$$\mathrm{L} = \int_0^{a'} x^u(a'-x)^{u+\frac{1}{2}}e^{b\sqrt{x(a'-x)}}\ {}^{p,q}\Phi^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(Zx^A(a'-x)^A, Tx^B(a'-x)^B, s, a)$$

$$I(Z_1x^C(a-x)^C, Z_2x^D(a-x)^D)\mathrm{d}x = \ 2^{-2u-1}\sqrt{\pi}a^{2u+\frac{3}{2}}\sum_{i,j,n'=0}^{\infty}\ {}^{p,q}A^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(s,a)\ \frac{1}{n'!}\left(\frac{a'b}{2}\right)^{n'}$$

$$\mathrm{Z}^iT^j\ 2^{-2Bi-2Cj}a^{2Bi+2Cj}\ \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\theta(s,t)\psi(s)\phi(t)Z_1^sZ_2^t\ \ 2^{2At}a'^{-2At}$$

$$\frac{\Gamma(2u+2Ai+2Bj+2Cs+2Dt+2)}{\Gamma(2u+2Bi+2Cj+2Cs+2Dt+\frac{5}{2})}\ \frac{(2u+2Ai+2Bj+2Cs+2Dt+2)_{n'}}{\Gamma(2u+2Ai+2Bj+2Cs+2Dt+\frac{5}{2})_{n'}}\ \ \mathrm{ds\ dt} \tag{3.5}$$

Now we appel the relation $\Gamma(a')(a')_n = \Gamma(a'+n)$, this gives :

$$\mathrm{L} = \int_0^{a'} x^u(a'-x)^{u+\frac{1}{2}}e^{b\sqrt{x(a'-x)}}\ {}^{p,q}\Phi^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(Zx^A(a'-x)^A, Tx^B(a'-x)^B, s, a)$$

$$I(Z_1x^C(a-x)^C, Z_2x^D(a-x)^D)\mathrm{d}x = 2^{-2u-1}\sqrt{\pi}a^{2u+\frac{3}{2}}\sum_{i,j,n'=0}^{\infty}\ {}^{p,q}A^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(s,a)\ \frac{n!}{\left(\frac{3}{2}\right)_{n'}(n+1)}\ \left(\frac{a'b}{2}\right)^{n'}$$

$$\mathrm{Z}^iT^j\ 2^{-2Bi-2Cj}a^{2Bi+2Cj}\ \ \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\theta(s,t)\psi(s)\phi(t)Z_1^sZ_2^t\ 2^{2At}a'^{-2At}$$

$$\frac{\Gamma\left(2u+2Ai+2Bj+2Cs+2Dt+2+n'\right)}{\Gamma\left(2u+2Ai+2Bj+2As+2Dt+\frac{5}{2}+n'\right)}\ \mathrm{ds\ dt} \tag{3.6}$$

We interpret this double Mellin-Barnes integrals contour of the modified generalized I-function of two variables, we obtain the desired result.

In the following section, we cite several particular cases of this I-function of two variables.

ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)

https://doi.org/10.31033/ijrasb.9.1.1

IV. SPECIAL CASES

First time, we consider the generalized I-functions of two variables which are special cases of the modified of generalized I-function of two variables. We have and

Corollary 1

$$\int_0^{a'} x^u(a'-x)^{u+\frac{1}{2}}e^{b\sqrt{x(a'-x)}}\mathrm{d}x\ {}^{p,q}\Phi^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(Zx^A(a'-x)^A, Tx^B(a'-x)^B, s, a)$$

$$I(Z_1x^C(a-x)^C, Z_2x^D(a-x)^D)\mathrm{d}x = 2^{-2u-1}\sqrt{\pi}a^{2s+\frac{3}{2}}\sum_{i,j,n'=0}^{\infty}\ {}^{p,q}A^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(s,a)\frac{1}{n'!}\left(\frac{a'b}{2}\right)^{n'}$$

$$Z^iT^j\,2^{-2Bi-2Cj}a^{2Bi+2Cj}\ I^{m_1,n_1+1:m_3,n_3:m_4,n_4}_{p_1+1,q_1+1:p_3,q_3:p_4,q_4}$$

$$\left(\begin{array}{c|c} Z_1\left(\frac{a'}{2}\right)^{2C} & \mathbf{A}_1,\{(a_i;\alpha_i,A_i;\mathbf{A}_i)\}_{1,p_1}:\{(e_i;E_i;\mathbf{E}_i)\}_{1,p_3},\{(g_i;G_i;\mathbf{G}_i)\}_{1,p_4} \\ \cdot & \cdot \\ \cdot & \cdot \\ Z_2\left(\frac{a'}{2}\right)^{2D} & \{(\mathrm{b}_i;\beta_i,B_i;\mathbf{B}_i)\}_{1,q_1},\mathbf{B}_1:\{(f_i;F_i;\mathbf{F}_i)\}_{1,q_3},\{(h_i;H_i;\mathbf{H}_i)\}_{1,q_4}\end{array}\right) \tag{4.1}$$

under the same conditions and notations that the theorem and $\mathrm{m}_2 = n_2 = p_2 = q_2 = 0$. We have

$|argz_1| < \frac{1}{2}U_1\pi, |argz_2| < \frac{1}{2}V_1\pi$, where U_1 and V_1 verify :

$$U_1 = \sum_{j=1}^{n_1}\mathbf{A}_j\alpha_j - \sum_{j=n_1+1}^{p_1}\mathbf{A}_j\alpha_j + \sum_{j=1}^{m_1}\beta_j\mathbf{B}_j - \sum_{j=m_1+1}^{q_1}\beta_j\mathbf{B}_j + \sum_{j=1}^{n_3}\mathbf{E}_jE_j - \sum_{j=n_3+1}^{p_3}\mathbf{E}_jE_j + \sum_{j=1}^{m_3}\mathbf{F}_jF_j - \sum_{j=m_3+1}^{q_3}\mathbf{F}_jF_j > 0 \tag{4.2}$$

$$V_1 = \sum_{j=1}^{n_1}\mathbf{A}_jA_j - \sum_{j=n_1+1}^{p_1}\mathbf{A}_jA_j + \sum_{j=1}^{m_1}B_j\mathbf{B}_j - \sum_{j=m_1+1}^{q_1}B_j\mathbf{B}_j + \sum_{j=1}^{n_4}\mathbf{G}_jG_j - \sum_{j=n_4+1}^{p_4}\mathbf{G}_jG_j + \sum_{j=1}^{m_4}\mathbf{H}_jH_j - \sum_{j=m_4+1}^{q_4}\mathbf{H}_jH_j > 0 \tag{4.3}$$

Now, we consider the above corollary with $m_1=0$, the function defined in the beginning reduces to I-function of two variables defined by Kumari et al. [14], this gives.

Corollary 2

$$\int_0^{a'} x^u(a'-x)^{u+\frac{1}{2}}e^{b\sqrt{x(a'-x)}}\mathrm{d}x\ {}^{p,q}\Phi^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(Zx^A(a'-x)^A, Tx^B(a'-x)^B, s, a)$$

$$I(Z_1x^C(a-x)^C, Z_2x^D(a-x)^D)\mathrm{d}x = 2^{-2u-1}\sqrt{\pi}a^{2s+\frac{3}{2}}\sum_{i,j,n'=0}^{\infty}\ {}^{p,q}A^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(s,a)\frac{1}{n'!}\left(\frac{a'b}{2}\right)^{n'}$$

$$Z^iT^j\,2^{-2Bi-2Cj}a^{2Bi+2Cj}\ I^{0,n_1+1:m_3,n_3:m_4,n_4}_{p_1+1,q_1+1:p_3,q_3:p_4,q_4}$$

$$\left(\begin{array}{c|c} Z_1\left(\frac{a'}{2}\right)^{2C} & \mathbf{A}_1,\{(a_i;\alpha_i,A_i;\mathbf{A}_i)\}_{1,p_1}:\{(e_i;E_i;\mathbf{E}_i)\}_{1,p_3},\{(g_i;G_i;\mathbf{G}_i)\}_{1,p_4} \\ \cdot & \cdot \\ \cdot & \cdot \\ Z_2\left(\frac{a'}{2}\right)^{2D} & \{(\mathrm{b}_i;\beta_i,B_i;\mathbf{B}_i)\}_{1,q_1},\mathbf{B}_1:\{(f_i;F_i;\mathbf{F}_i)\}_{1,q_3},\{(h_i;H_i;\mathbf{H}_i)\}_{1,q_4}\end{array}\right) \tag{4.4}$$

by respecting the conditions cited by the corollary 1and $\mathrm{m}_1 = 0$. This gives : $|argz_1| < \frac{1}{2}U_1'\pi, |argz_2| < \frac{1}{2}V_1'\pi$, U_1' where V_1'.

$$U_1' = \sum_{j=1}^{n_1}\mathbf{A}_j\alpha_j - \sum_{j=n_1+1}^{p_1}\mathbf{A}_j\alpha_j - \sum_{j=1}^{q_1}\beta_j\mathbf{B}_j + \sum_{j=1}^{n_3}\mathbf{E}_jE_j - \sum_{j=n_3+1}^{p_3}\mathbf{E}_jE_j + \sum_{j=1}^{m_3}\mathbf{F}_jF_j - \sum_{j=m_3+1}^{q_3}\mathbf{F}_jF_j > 0 \tag{4.5}$$

ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)

https://doi.org/10.31033/ijrasb.9.1.1

by respecting the conditions cited by the corollary 1and $m_1 = 0$. This gives : $|argz_1| < \frac{1}{2}U_1'\pi, |argz_2| < \frac{1}{2}V_1'\pi$, U_1' where V_1'.

$$U_1' = \sum_{j=1}^{n_1} \mathbf{A}_j\alpha_j - \sum_{j=n_1+1}^{p_1} \mathbf{A}_j\alpha_j - \sum_{j=1}^{q_1}\beta_j\mathbf{B}_j + \sum_{j=1}^{n_3}\mathbf{E}_jE_j - \sum_{j=n_3+1}^{p_3}\mathbf{E}_jE_j + \sum_{j=1}^{m_3}\mathbf{F}_jF_j - \sum_{j=m_3+1}^{q_3}\mathbf{F}_jF_j > 0 \tag{4.5}$$

$$V_1' = \sum_{j=1}^{n_1}\mathbf{A}_jA_j - \sum_{j=n_1+1}^{p_1}\mathbf{A}_jA_j - \sum_{j=1}^{q_1}B_j\mathbf{B}_j + \sum_{j=1}^{n_4}\mathbf{G}_jG_j - \sum_{j=n_4+1}^{p_4}\mathbf{G}_jG_j + \sum_{j=1}^{m_4}\mathbf{H}_jH_j - \sum_{j=m_4+1}^{q_4}\mathbf{H}_jH_j > 0 \tag{4.6}$$

The modified generalized I-function reduces to Modified generalized H-function defined by Prasad and Prasad [14]. we have : $(\mathbf{A_i} = \mathbf{B_i} = \mathbf{C_i} = \mathbf{D_i} = \mathbf{E_i} = \mathbf{F_i} = \mathbf{G_i} = \mathbf{H_i} = 1)$ and we get :

Corollary 3

$$\int_0^{a'} x^u(a'-x)^{u+\frac{1}{2}}e^{b\sqrt{x(a'-x)}}\mathrm{d}x \; {}^{p,q}\Phi^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(Zx^A(a'-x)^A, Tx^B(a'-x)^B, s, a)$$

$$H(Z_1x^C(a-x)^C, Z_2x^D(a-x)^D)\mathrm{d}x = 2^{-2u-1}\sqrt{\pi}a^{2s+\frac{3}{2}}\sum_{i,j,n'=0}^{\infty} \; {}^{p,q}A^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(s,a)$$

$$\frac{1}{n'!}\left(\frac{a'b}{2}\right)^{n'} \; Z^iT^j\, 2^{-2Bi-2Cj}a^{2Bi+2Cj}\, H^{m_1,n_1+2:m_2,n_2:m_3,n_3:m_4,n_4}_{p_1+2,q_1+2:p_2,q_2:p_3,q_3:p_4,q_4}$$

$$\left(\begin{array}{c|c} Z_1\left(\frac{a'}{2}\right)^{2C} & \mathbf{A}_1', \{(a_i;\alpha_i,A_i)\}_{1,p_1} : \{(c_i;\gamma_i,C_i)\}_{1,p_2} : \{(e_i;E_i)\}_{1,p_3}, \{(g_i;G_i)\}_{1,p_4} \\ \cdot & \cdot \\ \cdot & \cdot \\ Z_2\left(\frac{a'}{2}\right)^{2D} & \{(b_i;\beta_i,B_i)\}_{1,q_1}, \mathbf{B}_1' : \{(d_i;\delta_i,D_i)\}_{1,q_2} : \{(f_i;F_i)\}_{1,q_3}, \{(h_i;H_i)\}_{1,q_4} \end{array}\right) \tag{4.7}$$

with the same conditions and notations that theorem 1 and $(\mathbf{A_i} = \mathbf{B_i} = \mathbf{C_i} = \mathbf{D_i} = \mathbf{E_i} = \mathbf{F_i} = \mathbf{G_i} = \mathbf{H_i} = 1)$, where

$|argz_1| < \frac{1}{2}U_1\pi, |argz_2| < \frac{1}{2}V_1\pi$, U_1 and V_1 are defined by the following formulas :

$$U_1 = \sum_{j=1}^{n_1}\alpha_j - \sum_{j=n_1+1}^{p_1}\alpha_j + \sum_{j=1}^{m_1}\beta_j - \sum_{j=m_1+1}^{q_1}\beta_j + \sum_{j=1}^{n_2}\gamma_j - \sum_{j=n_2+1}^{p_2}\gamma_j + \sum_{j=1}^{m_2}\delta_j$$

$$- \sum_{j=m_2+1}^{q_2}\delta_j + \sum_{j=1}^{n_3}E_j - \sum_{j=n_3+1}^{p_3}E_j + \sum_{j=1}^{m_3}F_j - \sum_{j=m_3+1}^{q_3}F_j > 0 \tag{4.8}$$

$$V_1 = \sum_{j=1}^{n_1}A_j - \sum_{j=n_1+1}^{p_1}A_j + \sum_{j=1}^{m_1}B_j - \sum_{j=m_1+1}^{q_1}B_j + \sum_{j=1}^{n_2}C_j - \sum_{j=n_2+1}^{p_2}C_j + \sum_{j=1}^{m_2}D_j$$

$$- \sum_{j=m_2+1}^{q_2}\delta_j + \sum_{j=1}^{n_4}G_j - \sum_{j=n_4+1}^{p_4}G_j + \sum_{j=1}^{m_4}H_j - \sum_{j=m_4+1}^{q_4}H_j > 0 \tag{4.9}$$

where :

$$\mathbf{A}_1' = (-1-2u-2Ai-2Bj-n'; 2C, 2D); \mathbf{B}_1' = \left(-\tfrac{3}{2} - 2u - 2Ai - 2Bj; 2C, 2D\right) \tag{4.10}$$

We suppose $m_2 = n_2 = p_2 = q_2 = 0$, we obtain the generalized H-function of two variables and we have :

ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)

https://doi.org/10.31033/ijrasb.9.1.1

Corollary 4

$$\int_0^{a'} x^u (a'-x)^{u+\frac{1}{2}} e^{b\sqrt{x(a'-x)}} \mathrm{d}x \; {}^{p,q}\Phi^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(Zx^A(a'-x)^A, Tx^B(a'-x)^B, s, a)$$

$$H(Z_1x^C(a-x)^C, Z_2x^D(a-x)^D)\mathrm{d}x = 2^{-2u-1}\sqrt{\pi}a^{2s+\frac{3}{2}} \sum_{i,j,n'=0}^{\infty} \; {}^{p,q}A^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(s,a)$$

$$\frac{1}{n'!}\left(\frac{a'b}{2}\right)^{n'} \; Z^iT^j \, 2^{-2Bi-2Cj} a^{2Bi+2Cj} \, H^{m_1,n_1+2:m_3,n_3:m_4,n_4}_{p_1+2,q_1+2:p_3,q_3:p_4,q_4}$$

$$\left(\begin{array}{c|c} Z_1\left(\frac{a'}{2}\right)^{2C} & \mathbf{A}'_1, \{(a_i;\alpha_i,A_i)\}_{1,p_1} : \{(e_i;E_i)\}_{1,p_3}, \{(g_i;G_i)\}_{1,p_4} \\ \cdot & \cdot \\ \cdot & \cdot \\ Z_2\left(\frac{a'}{2}\right)^{2D} & \{(\mathrm{b}_i;\beta_i,B_i)\}_{1,q_1}, \mathbf{B}'_1 : \{(f_i;F_i)\}_{1,q_3}, \{(h_i;H_i)\}_{1,q_4} \end{array}\right) \tag{4.11}$$

By respecting the conditions and the same notations write in the above corollary and we have the following conditions where:

$|argz_1| < \frac{1}{2}U_1\pi, |argz_2| < \frac{1}{2}V_1\pi$, U_1 and V_1 are defined by the following formulas :

$$U_1 = \sum_{j=1}^{n_1}\alpha_j - \sum_{j=n_1+1}^{p_1}\alpha_j + \sum_{j=1}^{m_1}\beta_j - \sum_{j=m_1+1}^{q_1}\beta_j + \sum_{j=1}^{n_2}\gamma_j - \sum_{j=n_2+1}^{p_2}\gamma_j + \sum_{j=1}^{m_2}\delta_j$$

$$-\sum_{j=m_2+1}^{q_2}\delta_j + \sum_{j=1}^{n_3}E_j - \sum_{j=n_3+1}^{p_3}E_j + \sum_{j=1}^{m_3}F_j - \sum_{j=m_3+1}^{q_3}F_j > 0 \tag{4.12}$$

$$V_1 = \sum_{j=1}^{n_1}A_j - \sum_{j=n_1+1}^{p_1}A_j + \sum_{j=1}^{m_1}B_j - \sum_{j=m_1+1}^{q_1}B_j + \sum_{j=1}^{n_2}C_j - \sum_{j=n_2+1}^{p_2}C_j + \sum_{j=1}^{m_2}D_j$$

$$-\sum_{j=m_2+1}^{q_2}\delta_j + \sum_{j=1}^{n_4}G_j - \sum_{j=n_4+1}^{p_4}G_j + \sum_{j=1}^{m_4}H_j - \sum_{j=m_4+1}^{q_4}H_j > 0 \tag{4.13}$$

The quantities $\mathbf{A}'_1$ and $\mathbf{B}'_1$ are defined by the equation (4.10).

Taking $\mathrm{m}_1 = 0$, the generalized H-function of two variables reduces to H-function of two variables defined by Gupta and Mittal [12], then :

Corollary 5

$$\int_0^{a'} x^u (a'-x)^{u+\frac{1}{2}} e^{b\sqrt{x(a'-x)}} \mathrm{d}x \; {}^{p,q}\Phi^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(Zx^A(a'-x)^A, Tx^B(a'-x)^B, s, a)$$

$$H(Z_1x^C(a-x)^C, Z_2x^D(a-x)^D)\mathrm{d}x = 2^{-2u-1}\sqrt{\pi}a^{2s+\frac{3}{2}} \sum_{i,j,n'=0}^{\infty} \; {}^{p,q}A^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(s,a)$$

$$\frac{1}{n'!}\left(\frac{a'b}{2}\right)^{n'} \; Z^iT^j \, 2^{-2Bi-2Cj} a^{2Bi+2Cj} \, H^{0,n_1+2:m_3,n_3:m_4,n_4}_{p_1+2,q_1+2:p_3,q_3:p_4,q_4}$$

$$\left(\begin{array}{c|c} Z_1\left(\frac{a'}{2}\right)^{2C} & \mathbf{A}'_1, \{(a_i;\alpha_i,A_i)\}_{1,p_1} : \{(e_i;E_i)\}_{1,p_3}, \{(g_i;G_i)\}_{1,p_4} \\ \cdot & \cdot \\ \cdot & \cdot \\ Z_2\left(\frac{a'}{2}\right)^{2D} & \{(\mathrm{b}_i;\beta_i,B_i)\}_{1,q_1}, \mathbf{B}'_1 : \{(f_i;F_i)\}_{1,q_3}, \{(h_i;H_i)\}_{1,q_4} \end{array}\right) \tag{4.14}$$

ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)
https://doi.org/10.31033/ijrasb.9.1.1

under the same conditions and notations the above corollary and $m_1 = 0$, where

$|argz_1| < \frac{1}{2}U_1\pi, |argz_2| < \frac{1}{2}V_1\pi$, U_1 and V_1 are defined by the following formulas :

$$U_1 = \sum_{j=1}^{n_1}\alpha_j - \sum_{j=n_1+1}^{p_1}\alpha_j - \sum_{j=1}^{q_1}\beta_j + \sum_{j=1}^{n_2}\gamma_j - \sum_{j=n_2+1}^{p_2}\gamma_j + \sum_{j=1}^{m_2}\delta_j$$

$$- \sum_{j=m_2+1}^{q_2}\delta_j + \sum_{j=1}^{n_3}E_j - \sum_{j=n_3+1}^{p_3}E_j + \sum_{j=1}^{m_3}F_j - \sum_{j=m_3+1}^{q_3}F_j > 0 \tag{4.15}$$

$$- \sum_{j=m_2+1}^{q_2}\delta_j + \sum_{j=1}^{n_4}G_j - \sum_{j=n_4+1}^{p_4}G_j + \sum_{j=1}^{m_4}H_j - \sum_{j=m_4+1}^{q_4}H_j > 0 \tag{4.16}$$

The generalized modified H-function reduces to the generalized modified of G-function of two variables defined by Agarwal [1], we suppose : $D = C = 1$, and we have the conditions :
$(\alpha_i)_{1,p_1} = (A_i)_{1,p_1} = (\gamma_i)_{1,p_2} = (C_i)_{1,p_2} = (E_i)_{1,p_3} = (G_i)_{1,p_4} = 1 = (\beta_i)_{1,q_1} = (B_i)_{1,q_1} = (\delta_i)_{1,q_2} = (D_i)_{1,q_2} = (F_i)_{1,q_3} = (H_i)_{1,q_4}$
, we have the result :

Corollary 6

$$\int_0^{a'} x^u(a'-x)^{u+\frac{1}{2}}e^{b\sqrt{x(a'-x)}}\mathrm{d}x \; {}^{p,q}\Phi^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(Zx^A(a'-x)^A, Tx^B(a'-x)^B, s, a)$$

$$G(Z_1x(a-x), Z_2x(a-x))\mathrm{d}x = 2^{-2u-1}\sqrt{\pi}a^{2s+\frac{3}{2}}\sum_{i,j,n'=0}^{\infty} \; {}^{p,q}A^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(s,a)$$

$$\frac{1}{n'!}\left(\frac{a'b}{2}\right)^{n'} \; Z^iT^j\, 2^{-2Bi-2Cj}a^{2Bi+2Cj} \; G^{m_1,n_1+2:m_2,n_2:m_3,n_3:m_4,n_4}_{p_1+2,q_1+2:p_2,q_2:p_3,q_3:p_4,q_4}$$

$$\left(\begin{array}{c|c} Z_1 & A_1', (a_1)_{1,p_1} : (c_i)_{1,p_2} : (e_i)_{1,p_3}, (g_i)_{1,p_4} \\ \cdot & \cdot \\ \cdot & \cdot \\ Z_2 & (b_i)_{1,q_1}, B_1' : (d_i)_{1,q_2} : (f_i)_{1,q_3}, (h_i)_{1,q_4} \end{array}\right) \tag{4.17}$$

where

$$A_1' = (-1-2u-2Ai-2Bj-n'); B_1' = \left(-\tfrac{3}{2}-2u-2Ai-2Bj\right) \tag{4.18}$$

and $|argZ_1| < \frac{1}{2}U_1\pi, |argZ_2| < \frac{1}{2}V_1\pi$, U_1, V_1 are defined by the following formulas :

$$U_1 = \left[m_1+n_1+m_2+n_2+m_3+n_3-\tfrac{1}{2}(p_1+q_1+p_2+q_2+p_3+q_3)\right] \tag{4.19}$$

and

$$V_1 = \left[m_1+n_1+m_2+n_2+m_4+n_4-\tfrac{1}{2}(p_1+q_1+p_2+q_2+p_4+q_4)\right] \tag{4.20}$$

We consider the generalized Meijer G-function, then $m_2 = n_2 = p_2 = q_2 = 0$, this gives :

$(\alpha_i)_{1,p_1} = (A_i)_{1,p_1} = (E_i)_{1,p_3} = (G_i)_{1,p_4} = 1 = (\beta_i)_{1,q_1} = (B_i)_{1,q_1} = (F_i)_{1,q_3} = (H_i)_{1,q_4}$ and we have :

Corollary 7

$$\int_0^{a'} x^u(a'-x)^{u+\frac{1}{2}}e^{b\sqrt{x(a'-x)}}\mathrm{d}x \; {}^{p,q}\Phi^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(Zx^A(a'-x)^A, Tx^B(a'-x)^B, s, a)$$

$$G(Z_1x(a-x), Z_2x(a-x))\mathrm{d}x = 2^{-2u-1}\sqrt{\pi}a^{2s+\frac{3}{2}}\sum_{i,j,n'=0}^{\infty} \; {}^{p,q}A^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(s,a)$$

ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)

https://doi.org/10.31033/ijrasb.9.1.1

$$\frac{1}{n'!}\left(\frac{a'b}{2}\right)^{n'} \quad Z^iT^j\, 2^{-2Bi-2Cj}a^{2Bi+2Cj}\; G^{m_1,n_1+2:m_3,n_3:m_4,n_4}_{p_1+2,q_1+2:p_3,q_3:p_4,q_4}\left(\begin{array}{c|c} Z_1 & A_1', (a_1)_{1,p_1} : (e_i)_{1,p_3}, (g_i)_{1,p_4} \\ \cdot & \cdot \\ \cdot & \cdot \\ Z_2 & (b_i)_{1,q_1}, B_1' : (f_i)_{1,q_3}, (h_i)_{1,q_4} \end{array}\right) \tag{4.21}$$

Our function cited above reduces to generalized Meijer G-function of two variables dfined by Agarwal [1], In this situation, we take $m_1 = 0$, and $(\alpha_i)_{1,p_1} = (A_i)_{1,p_1} = (E_i)_{1,p_3} = (G_i)_{1,p_4} = 1 = (\beta_i)_{1,q_1} = (B_i)_{1,q_1} = (F_i)_{1,q_3} = (H_i)_{1,q_4}$ and and $|argZ_1| < \frac{1}{2}U_1\pi, |argZ_2| < \frac{1}{2}V_1\pi$, U_1, V_1 are defined by the following formulas :

$$U_1 = \left[m_1 + n_1 + m_3 + n_3 - \tfrac{1}{2}(p_1 + q_1 + p_3 + q_3)\right] \tag{4.22}$$

and

$$V_1 = \left[m_1 + n_1 + m_4 + n_4 - \tfrac{1}{2}(p_1 + q_1 + p_4 + q_4)\right] \tag{4.23}$$

We consider the above situation, we take $m_1 = 0$,

Corollary 8

$$\int_0^{a'} x^u(a'-x)^{u+\frac{1}{2}}e^{b\sqrt{x(a'-x)}}\mathrm{d}x\; {}^{p,q}\Phi^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(Zx^A(a'-x)^A, Tx^B(a'-x)^B, s, a)$$

$$G(Z_1x(a-x), Z_2x(a-x))\mathrm{d}x = 2^{-2u-1}\sqrt{\pi}a^{2s+\frac{3}{2}}\sum_{i,j,n'=0}^{\infty}\; {}^{p,q}A^{\alpha:\beta;\gamma;\lambda,\mu:\upsilon;\rho;\sigma}_{H:J;K;L;M,N:R;V;W}(s,a)$$

$$\frac{1}{n'!}\left(\frac{a'b}{2}\right)^{n'} \quad Z^iT^j\, 2^{-2Bi-2Cj}a^{2Bi+2Cj}\; G^{0,n_1+2:m_3,n_3:m_4,n_4}_{p_1+2,q_1+2:p_3,q_3:p_4,q_4}\left(\begin{array}{c|c} Z_1 & A_1', (a_1)_{1,p_1} : (e_i)_{1,p_3}, (g_i)_{1,p_4} \\ \cdot & \cdot \\ \cdot & \cdot \\ Z_2 & (b_i)_{1,q_1}, B_1' : (f_i)_{1,q_3}, (h_i)_{1,q_4} \end{array}\right) \tag{4.24}$$

with and $|argZ_1| < \frac{1}{2}U_1\pi, |argZ_2| < \frac{1}{2}V_1\pi$, U_1, V_1 are defined by the following formulas :

$$U_1 = \left[n_1 + m_3 + n_3 - \tfrac{1}{2}(p_1 + q_1 + p_3 + q_3)\right] \tag{4.25}$$

and

$$V_1 = \left[n_1 + m_4 + n_4 - \tfrac{1}{2}(p_1 + q_1 + p_4 + q_4)\right] \tag{4.26}$$

Now, we consider the modified of generalized I-function of two variables and we give the particular cases of the generalized of extended Hurwitz's Zeta function of two variables. We will use the notations of the first section.

V. SPECIAL CASES OF THE EXTENTED ZETA FUNCTION OF TWO VARIABLES

Let J = R = 0, we get

Corollary 9

$$\int_0^{a'} x^u(a'-x)^{u+\frac{1}{2}}e^{b\sqrt{x(a'-x)}}\; {}^{p,q}\Phi^{\alpha:\gamma;\lambda,\mu:\rho;\sigma}_{H:K;L;M,N:V;W}(Zx^A(a'-x)^ATx^B(a'-x)^B, s, a)$$

$$I(Z_1x^C(a-x)^C, Z_2x^D(a-x)^D)\mathrm{d}x = 2^{-2u-1}\sqrt{\pi}a^{2u+\frac{3}{2}}\sum_{i,j,n'=0}^{\infty}\; {}^{p,q}A^{\alpha:\gamma;\lambda,\mu:\sigma}_{H:K;L;M,N:V;W}(s,a)\frac{1}{n'!}\left(\frac{a'b}{2}\right)^{n'}$$

$$Z^iT^j\, 2^{-2Ai-2Bj}a^{2Ai+2Bj}\; I^{m_1,n_1+1:m_2,n_2:m_3,n_3:m_4,n_4}_{p_1+1,q_1+1:p_2,q_2:p_3,q_3:p_4,q_4}$$

ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)

https://doi.org/10.31033/ijrasb.9.1.1

$$\left(\begin{array}{c|c} Z_1\left(\frac{a'}{2}\right)^{2C} & \mathbf{A}_1,\{(a_i;\alpha_i,A_i;\mathbf{A}_i)\}_{1,p_1}:\{(c_i;\gamma_i,C_i;\mathbf{C}_i)\}_{1,p_2}:\{(e_i;E_i;\mathbf{E}_i)\}_{1,p_3},\{(g_i;G_i;\mathbf{G}_i)\}_{1,p_4} \\ \cdot & \cdot \\ \cdot & \cdot \\ Z_2\left(\frac{a'}{2}\right)^{2D} & \{(\mathrm{b}_i;\beta_i,B_i;\mathbf{B}_i)\}_{1,q_1},\mathbf{B}_1:\{(d_i;\delta_i,D_i;\mathbf{D}_i)\}_{1,q_2}:\{(f_i;F_i;\mathbf{F}_i)\}_{1,q_3},\{(h_i;H_i;\mathbf{H}_i)\}_{1,q_4} \end{array}\right) \tag{5.1}$$

under the conditions verified by the theorem and we replace the conditions (E) by the conditions (E') defined in the section 1.

Taking $\mathrm{H}=\mathrm{N}=0$, this gives :

Corollary 10

$$\int_0^{a'} x^{\mathbf{s}}(a'-x)^{\mathbf{s}+\frac{1}{2}}e^{b\sqrt{x(a'-x)}}\ {}_{p,q}\Phi^{\beta;\gamma;\lambda,\upsilon:\rho;\sigma}_{K;L;M,N:V;W}(Zx^A(a'-x)^A Tx^B(a'-x)^B,s,a)$$

$$I(Z_1x^C(a-x)^C,Z_2x^D(a-x)^D)\mathrm{d}x=2^{-2\mathbf{s}-1}\sqrt{\pi}a^{2\mathbf{s}+\frac{3}{2}}\sum_{i,j,n'=0}^{\infty}\ {}_{p,q}A^{\beta;\gamma;\lambda:\upsilon;\rho;\sigma}_{K;L;M:R;V;W}(s,a)\frac{1}{n'!}\left(\frac{a'b}{2}\right)^{n'}$$

$$Z^iT^j\,2^{-2Bi-2Cj}a^{2Bi+2Cj}\,I^{m_1,n_1+1:m_2,n_2:m_3,n_3:m_4,n_4}_{p_1+1,q_1+1:p_2,q_2:p_3,q_3:p_4,q_4}$$

$$\left(\begin{array}{c|c} Z_1\left(\frac{a'}{2}\right)^{2C} & \mathbf{A}_1,\{(a_i;\alpha_i,A_i;\mathbf{A}_i)\}_{1,p_1}:\{(c_i;\gamma_i,C_i;\mathbf{C}_i)\}_{1,p_2}:\{(e_i;E_i;\mathbf{E}_i)\}_{1,p_3},\{(g_i;G_i;\mathbf{G}_i)\}_{1,p_4} \\ \cdot & \cdot \\ \cdot & \cdot \\ Z_2\left(\frac{a'}{2}\right)^{2D} & \{(\mathrm{b}_i;\beta_i,B_i;\mathbf{B}_i)\}_{1,q_1},\mathbf{B}_1:\{(d_i;\delta_i,D_i;\mathbf{D}_i)\}_{1,q_2}:\{(f_i;F_i;\mathbf{F}_i)\}_{1,q_3},\{(h_i;H_i;\mathbf{H}_i)\}_{1,q_4} \end{array}\right) \tag{5.2}$$

Verifying conditions cited in the theorem and we replace the conditions (E) by the conditions (E'') named in the first section.

Now, we consider the generalized Hurwitz-Lerch zeta-function of two variables defined by Pathan and Daman [15], we have :

Corollary 11

$$\int_0^{a'} x^{u}(a'-x)^{u+\frac{1}{2}}e^{b\sqrt{x(a'-x)}}\ \Phi^{p,q}_{\alpha;\beta;\gamma:\lambda,\mu;\upsilon}(Zx^A(a'-x)^A,Tx^B(a'-x)^B,s,a)$$

$$I(Z_1x^C(a-x)^C,Z_2x^D(a-x)^D)\mathrm{d}x=2^{-2\mathbf{s}-1}\sqrt{\pi}a^{2\mathbf{s}+\frac{3}{2}}\sum_{i,j,n'=0}^{\infty}\ \frac{1}{n'!}\left(\frac{a'b}{2}\right)^{n'}\ \mathrm{A}^{p,q}_{\alpha;\beta;\gamma:\lambda,\mu;\upsilon}(s,a)$$

$$Z^iT^j\,2^{-2Bi-2Cj}a^{2Bi+2Cj}\,I^{m_1,n_1+1:m_2,n_2:m_3,n_3:m_4,n_4}_{p_1+1,q_1+1:p_2,q_2:p_3,q_3:p_4,q_4}$$

$$\left(\begin{array}{c|c} Z_1\left(\frac{a'}{2}\right)^{2C} & \mathbf{A}_1,\{(a_i;\alpha_i,A_i;\mathbf{A}_i)\}_{1,p_1}:\{(c_i;\gamma_i,C_i;\mathbf{C}_i)\}_{1,p_2}:\{(e_i;E_i;\mathbf{E}_i)\}_{1,p_3},\{(g_i;G_i;\mathbf{G}_i)\}_{1,p_4} \\ \cdot & \cdot \\ \cdot & \cdot \\ Z_2\left(\frac{a'}{2}\right)^{2D} & \{(\mathrm{b}_i;\beta_i,B_i;\mathbf{B}_i)\}_{1,q_1},\mathbf{B}_1:\{(d_i;\delta_i,D_i;\mathbf{D}_i)\}_{1,q_2}:\{(f_i;F_i;\mathbf{F}_i)\}_{1,q_3},\{(h_i;H_i;\mathbf{H}_i)\}_{1,q_4} \end{array}\right) \tag{5.3}$$

By considering the conditions and notations of the theorem and the following conditions $\gamma,\upsilon,a\notin 0,-1,-2,\cdots\}s\in\mathbb{C},p,q,Re(s)>0$ when $|z|,|t|<1$ and $\mathrm{Re}(\gamma+\upsilon+s-\alpha-\beta-\lambda-\mu)>0$ when $|z|=|t|=1$ are satisfied instead of the conditions (E).

We consider the generalized Hurwitz-Lerch zeta-function studied by Bin-Saad, Garg et al. [5,6], we obtain :

ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)

https://doi.org/10.31033/ijrasb.9.1.1

Corollary 12

$$\int_0^{a'} x^u (a'-x)^{u+\frac{1}{2}} e^{b\sqrt{x(a'-x)}}\, \phi(Zx^B(a'-x)^B, s, a) I(Z_1 x^C (a-x)^C, Z_2 x^D (a-x)^D) \mathrm{d}x$$

$$= 2^{-2\mathbf{s}-1}\sqrt{\pi} a^{2\mathbf{s}+\frac{3}{2}} \sum_{i,n'=0}^{\infty} \frac{1}{n'!}\left(\frac{a'b}{2}\right)^{n'} \frac{Z^i}{(a+s)^i}\; 2^{-2Bi} a^{2Bi}\, I^{m_1,n_1+1:m_2,n_2:m_3,n_3:m_4,n_4}_{p_1+1,q_1+1:p_2,q_2:p_3,q_3:p_4,q_4}$$

$$\left(\begin{array}{c|c} \mathrm{Z}_1 \left(\frac{a'}{2}\right)^{2C} & \mathbf{A}_1, \{(a_i;\alpha_i, A_i;\mathbf{A}_i)\}_{1,p_1} : \{(c_i;\gamma_i, C_i;\mathbf{C}_i)\}_{1,p_2} : \{(e_i;E_i;\mathbf{E}_i)\}_{1,p_3}, \{(g_i;G_i;\mathbf{G}_i)\}_{1,p_4} \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{Z}_2 \left(\frac{a'}{2}\right)^{2D} & \{(\mathrm{b}_i;\beta_i, B_i;\mathbf{B}_i)\}_{1,q_1}, \mathbf{B}_1 : \{(d_i;\delta_i, D_i;\mathbf{D}_i)\}_{1,q_2} : \{(f_i;F_i;\mathbf{F}_i)\}_{1,q_3}, \{(h_i;H_i;\mathbf{H}_i)\}_{1,q_4} \end{array}\right) \quad (5.4)$$

with the conditions mentionned by the theorem and we have $\mu \in \mathbb{C}, a \in \mathbb{C}\setminus\mathbb{Z}^0; s \in \mathbb{C}$ when $|z|<1, Re(s-\mu)>1$ when $|z|=1$ instead the conditions (E) mentionned in he section1.

Let the generalized Hurwitz's Zeta function $\phi(x,s,a)$ is defined by [9, p.19] , this gives :

Corollary 13

$$\int_0^{a'} x^u (a'-x)^{u+\frac{1}{2}} e^{b\sqrt{x(a'-x)}}\, \phi^*(ZX^B(a'-x)^B, s, a)\;\; I(Z_1 x^C (a-x)^C, Z_2 x^D (a-x)^D) \mathrm{d}x = 2^{-2\mathbf{s}-1}\sqrt{\pi} a^{2\mathbf{s}+\frac{3}{2}}$$

$$\sum_{i,n'=0}^{\infty} \frac{1}{n'!}\left(\frac{a'b}{2}\right)^{n'} \mathrm{A}^*(i,s,a)\;\; \mathrm{Z}^i\, 2^{-2Bi} a^{2Bi}\;\; I^{m_1,n_1+1:m_2,n_2:m_3,n_3:m_4,n_4}_{p_1+1,q_1+1:p_2,q_2:p_3,q_3:p_4,q_4}$$

$$\left(\begin{array}{c|c} \mathrm{Z}_1 \left(\frac{a'}{2}\right)^{2C} & \mathbf{A}_1, \{(a_i;\alpha_i, A_i;\mathbf{A}_i)\}_{1,p_1} : \{(c_i;\gamma_i, C_i;\mathbf{C}_i)\}_{1,p_2} : \{(e_i;E_i;\mathbf{E}_i)\}_{1,p_3}, \{(g_i;G_i;\mathbf{G}_i)\}_{1,p_4} \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{Z}_2 \left(\frac{a'}{2}\right)^{2D} & \{(\mathrm{b}_i;\beta_i, B_i;\mathbf{B}_i)\}_{1,q_1}, \mathbf{B}_1 : \{(d_i;\delta_i, D_i;\mathbf{D}_i)\}_{1,q_2} : \{(f_i;F_i;\mathbf{F}_i)\}_{1,q_3}, \{(h_i;H_i;\mathbf{H}_i)\}_{1,q_4} \end{array}\right) \quad (5.5)$$

With the conditions using by the theorem are verified where $\mathrm{a} \in \mathbb{C}\setminus\mathbb{Z}^0; s \in \mathbb{C}$ when $|z|<1, Re(s)>1$ and $|z|=1$.

Remarks

We have the same generalized finite integral with the modified generalized of I-function of two variables defined by Kumari et al. [14], see Singh and Kumar for more details [22] and the special cases mentionned later and the Fox's H function, We have the same generalized multiple finite integrals with the incomplete aleph-function defined by Bansal et al. [3], the incomplete I-function studied by Bansal and Kumar. [2] and the incomplete Fox's H-function given by Bansal et al. [4], the Psi function defined by Pragathi et al. [17].

We have the same generalized finite integrals involving the extension of the generalized Hurwitz-Lerch zeta-function of two variables with the aleph-function defined by Sudland [27], the I-function defined by Saxena [20], by Rathie [19] and the Fox's H-function.

VI. CONCLUSION

The importance of our all the results lies in their manifold generality. First by specializing the various parameters as well as variable of the generalized modified I-function of two variables, we obtain a large number of results involving remarkably wide variety of useful special functions (or product of such special functions) which are expressible in terms of I-function of two variables or one variable defined by Rathie [19], H-function of two or one variables, Meijer's G-function of two or one variables and hypergeometric function of two or one variables. Secondly, by specializing the parameters of this unified finite integral, we can get a large number of integrals involving the modified generalized I-functions of two variables and the others functions seen in this document. Thirdly, by specializing the parameters of the variables of the Extented Zeta-function of two variables, we get a big number of known and news integrals.

REFERENCES

[1] R.P. Agarwal, "An extension of Meijer's G-function," Proc. Nat. Inst. Sci. India Part A, 31 (1965), 536-546.

ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)
https://doi.org/10.31033/ijrasb.9.1.1

[2] M.K. Bansal and D. Kumar, On the integral operators pertaining to a family of incomplete I-functions, AIM Mathematics 5(2) (2020), 1247-1259.
[3] M.K. Bansal, D. Kumar, K.S. Nisar and J. Singh, Certain fractional calculus and integral transform results of incomplete Aleph-functions with applications, Math. Mech; Appli. Sci (Wiley), (2020), 1-13.
[4] M.K. Bansal, D. Kumar, I. Khan, J. Singh and K.S. Nisar, Certain unified integrals associated with product of Mseries and incomplete H-functions, Mathematics, (7) (2019), 1-11.
[5] G. Maged G. Bin-Saad and Amani M. Hanballa, On hypergeometric series associated with the generalized zeta functions, SCIREA Journal of Mathematics 1 (2016), 53-62.
[6] Maged G. Bin-Saad, M. A. Pathan and Ali Z. Bin-Alhag, On multiple zeta function and associated properties, Turkish Journal of Analysis and Number Theory 6 (2018), 84-89
[7] B.L.J. Braaksma, Asymptotics expansions and analytic continuations for a class of Barnes-integrals, Compositio Math. 15 (1962-1964), 239-341.
[8] Y.A. Brychkov, Handbook of special functions, Derivatives, Integrals, Series and oher formulas, CRC. Press. Taylor and Francis Group. Boca. Raton, London, New York 2008.
[9] A. Erdélyi, W. Magnus, F. Oberhettinger and F. G. Tricomi, Higher Transcendental Functions, Vol. I, McGraw-Hill Book Inc., New York, Toronto and London, 1953.
[10] H. Exton, A note on a hypergeometric transformation, Bull. Calcutta Math. Soc. 71 (1979), 337-340.
[11] H. Exton, Reducible double hypergeometric functions and associated integrals, An. Fac. Ci. Univ. Porto 63(1-4) (1982), 137-143.
[12] K.C. Gupta, and P.K. Mittal, Integrals involving a generalized function of two variables, (1972), 430-437.
[13] D. Kumar, Generalized fractional differintegral operators of the Aleph-function of two variables, Journal of Chemical, Biological and Physical Sciences, Section C, 6(3) (2016), 1116-1131.
[14] K S. Kumari, T.M. Vasudevan Nambisan and A.K. Rathie, A study of I-functions of two variables, Le matematiche 69(1) (2014), 285-305.
[15] M. A. Pathan and O. Daman, On Generalization of Hurwitz zeta function, Non-Linear World Journal, (2018), to appear.
[16] M. A. Pathan, Maged G. Bin-Saad and J. A. Younis, Hurwitz Zeta Function of Two Variables and Associated Properties, Earthline Journal of Mathematical Sciences, Vol 3 (2), (2020), 297-315.
[17] Y. Pragathi Kumar and B. Satyanarayana, A study of Psi-function, Journal of Informatics and mathematical Sciences, Vol. 12 (2) (2020), 159-171.
[18] Y.N. Prasad and S. Prasad, (1979 -1980): Journal of scientific research, Banaras Hindu University, 30 (1), 67 -76.
[19] A.K. Rathie, A new generalization of generalized hypergeometric functions, Le matematiche 52 (2) (1997), 297-310.
[20] V.P. Saxena, The I-function, Anamaya Publishers, New Delhi, 2008.
[21] K. Sharma, On the Integral Representation and Applications of the Generalized Function of Two Variables, International Journal of Mathematical Engineering and Science, 3(1) (2014), 1-13.
[22] H. Singh and P. Kumar, Finite integral formulas involving multivariable Mittag-Leffler function and Modified I function, Int. Jr. of Mathematical sciences and Application, Vol 8(2), (2018), 115-128.
[23] H. M. Srivastava and J. Choi, Zeta and q-Zeta Functions and Associate Series and Integrals, Elsevier Science, Publishers, Amsterdam, London and New York, 2012.
[24] H. M. Srivastava and R. Panda, An integral representation for the product of two Jacobi polynomials, J. London Math. Soc. 12 (1976), 419-425.
[25] H. M. Srivastava and M. A. Pathan, Some bilateral generating functions for the extended Jacobi polynomials, Comment. Math. Uni. St. Paul 28(1) (1979), 23-30.
[26] N. Südland, N B. Baumann and T.F. Nonnenmacher, Open problem : who knows about the Aleph-functions? Fract. Calc. Appl. Anal., 1(4) (1998): 401-402